Solutions to Seminar exercise 5

1.First-order conditions for the basic monopoly model

$$\frac{\partial L}{\partial e_t^H} = p_t'(e_t^H)e_t^H + p_t(e_t^H) - \lambda_t \leq 0 \ (= 0 \text{ for } e_t^H > 0)$$
$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \ (= 0 \text{ for } R_t > 0)$$
$$\lambda_t \geq 0 \ (0 = \text{for } R_t < R_{t-1} + w_t - e_t^H)$$
$$\gamma_t \geq 0 \ (0 = \text{for } R_t < \bar{R}) \ , \qquad t = 1,..,T$$

Assuming in general i) electricity production positive in each period, ii) electricity price positive in each period.

Result 1: No change in the principle for change in water value, but under monopoly the price change may be independent of water value change.

Introducing the demand *flexibility* $\breve{\eta}_t = p_t' e_t^H / p_t$; the inverse of demand *elasticity*.

Using the flexibility-corrected price:

$$p_t(e_t^H)(1 + \breve{\eta}_t) = \lambda_t$$

NB! General monopoly knowledge: can only have a unique solution if the flexibility correction factor is greater or equal to zero: $(1 + \breve{\eta}_t) \geq 0$.

Result 2: monopoly leads to a change in the allocation of water on periods; the flexibility-corrected price is set equal to the water value implying more water is used in a period with a relatively low absolute value of the flexibility relative to a period with a higher absolute value.

Result 3: Without spilling the total production is constant, it is only the allocation on periods that change.

Result 4: Consider only two periods, and assume that in the social planning solution transfer of water from the relative demand-elastic period is period 1 and the relative demand-inelastic period is period 2. Furthermore, assume that the demand functions intersect to the left of the reservoir capacity measured for period 1 from the right-hand limit on available water in period 1 to the left. Then the monopoly solutions may lead to the reservoir constraint changing from being binding in the social planner solution to being non-binding in the monopoly case.

Result 5: Spilling may be optimal in a period if the optimal water value becomes zero and all locked-in water is not utilised at that output level; $p_t(e_t^H)(1+\breve{\eta}_t)=0, e_t^H < R_{t-1}+w_t-R_t$

Result 6: Consider the two-period case. If the reservoir constraint becomes binding and spilling is not optimal, then the monopoly solution for quantities and consumer prices becomes identical to the social solution.

Monopolist with hydro and thermal capacity

$$\frac{\partial L}{\partial e_t^H} = p_t'(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) + p_t(e_t^H + e_t^{Th}) - \lambda_t \le 0$$
$$(=0 \text{ for } e_t^H > 0)$$
$$\frac{\partial L}{\partial e_t^{Th}} = p_t'(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) + p_t(e_t^H + e_t^{Th}) - c'(e_t^{Th}) - \theta_t \le 0$$
$$(=0 \text{ for } e_t^{Th} > 0)$$
$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \; (=0 \text{ for } R_t > 0)$$
$$\lambda \ge 0 \; (=0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$
$$\theta_t \ge 0 \; (=0 \text{ for } e_t^{Th} < \bar{e}^{Th})$$
$$\gamma_t \ge 0 \; (0 = \text{ for } R_t < \bar{R})$$

Assume both hydro and thermal production to be positive.

Result 7: The flexibility-corrected price is set equal to the water value equal to the marginal cost of thermal (plus shadow price on capacity constraint if constraint is binding. The implication is that thermal marginal cost is lower than price; $p_t(x_t)(1+\breve{\eta}_t) = \lambda_t = c'(e_t^{Th}) + \theta_t$

Result 8: Use of thermal may be constant, but the prices vary.

The trade model:

$$\frac{\partial L}{\partial e_t^H} = p_t'(e_t^H - e_t^{XI})(e_t^H - e_t^{XI}) + p_t(e_t^H - e_t^{XI}) - \lambda_t \leq 0$$
$$(= 0 \text{ for } e_t^H > 0)$$
$$\frac{\partial L}{\partial e_t^{XI}} = -p_t'(e_t^H - e_t^{XI})(e_t^H - e_t^{XI}) - p_t(e_t^H - e_t^{XI}) - \alpha_t + \beta_t + p_t^{XI} = 0$$
$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \ (= 0 \text{ for } R_t > 0)$$
$$\lambda_t \geq 0 \ (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$
$$\gamma_t \geq 0 \ (= 0 \text{ for } R_t < \bar{R})$$
$$\alpha_t \geq 0 \ (= 0 \text{ for } e_t^{XI} < \bar{e}^{XI})$$
$$\beta_t \geq 0 \ (= 0 \text{ for } e_t^{XI} < -\bar{e}^{XI})$$

Assume positive production in all periods.

Result 9: The flexibility-corrected price is equal to the water value and equal to the import/export price minus (plus) the shadow price on the interconnector capacity if it is binding when the monopolist is exporting (importing); $p_t(e_t^H - e_t^{XI})(1 + \breve{\eta}_t) = \lambda_t = p_t^{XI} - \alpha_t (+\beta_t)$

Implication: the home price is higher than the import price (plus shadow price on interconnector) if importing (water value lower than price and water value equal to import price plus shadow price), and higher than export price if exporting.

Dominant hydro firm with a competitive thermal fringe

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^{Th}) + p_t'(e_t^H + e_t^{Th})e_t^H(1 + f'(e_t^H)) - \lambda_t \leq 0$$
$$(= 0 \text{ for } e_t^H > 0)$$
$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \ (= 0 \text{ for } R_t > 0)$$
$$\lambda_t \geq 0 \ (0 = \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$
$$\gamma_t \geq 0 \ (0 = \text{ for } R_t < \bar{R}) , \qquad t = 1,..,T$$

Assume positive production in all periods both for the dominant hydro firm and the thermal competitive fringe.

Fringe reaction follows from setting marginal cost equal to price; $p_t(x_t) = c'(e_t^{Th})$. The dominant firm must calculate in the quantity reaction of the fringe; $e_t^{Th} = f(e_t^H), f' < 0$

Result 10: Price equal to the marginal cost of the competitive fringe

Result 11: The conditional marginal revenue is closer to the demand function than the unconditional marginal revenue both due to a market share less than 100 % and the quantity reaction of the fringe moving in the opposite direction of the changes made by the dominant firm and counteracting the change in price, implying market price closer to conditional marginal revenue.

$$MR_{t|p'_t=c'} = p_t(1+\breve{\eta}_t \frac{e_t^H}{e_t^H + e_t^{Th}}) + p'_t f'(e_t^H)e_t^H \, , t = 1,..,T$$

Discovering use of market power:

Use the results for the different market situations to choose variables to observe.

Table. Detecting use of market power

Market structure	**Observable variables related to use of market power**	**Key equations**
Hydro monopolist	Degree of filling: never full reservoir handed over to next period Prices may change but reservoir constraints inactive Spilling of water	$p_t(e_t^H)(1+\breve{\eta}_t) = \lambda_t$ $p_t(e_t^H)(1+\breve{\eta}_t) = 0$
Hydro with thermal	Price higher than thermal marginal cost Use of thermal remains constant even if prices vary and water values constant	$p_t(x_t)(1+\breve{\eta}_t) = \lambda_t =$ $c'(e_t^{Th}) + \theta_t$
Hydro and trade	Home price higher than import/export price even if interconnector capacity is not constrained	$p_t(e_t^H - e_t^{XI})(1+\breve{\eta}_t) = \lambda_t =$ $p_t^{XI} - \alpha_t (+\beta_t)$
Dominant hydro with thermal competitive fringe	Outputs of dominant firm and competitive fringe move in opposite directions Prices may change but reservoir constraints inactive	$p_t(e_t^H + e_t^{Th}) = c'(e_t^{Th}) \Rightarrow$ $e_t^{Th} = f(e_t^H), f' < 0$

2. If the demand flexibility is constant (Cobb - Douglas demand function) = -0.02, the demand flexibility is -50, implying that a unique monopoly solution cannot be obtained!